Berner Fachhochschule
Haute école spécialisée bernoise
Bern University of Applied Sciences

CS Basics
1) Bases 2, 4, 8, 16, etc.

E. Benoist & C. Grothoff

Fall Term 2018-19

Computer Science Division

Bases

- Martian base
- Octal
- Hexadecimal
  - Conversions
  - Arithmetic in hexadecimal
- Binary
  - Hex as shorthand for binary

Martian base

We count in base 10

- **Signs used for counting**
  - 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0
  - a number is a list of signs 123 means $1 \times 100 + 2 \times 10 + 3$.
- **Other bases were used over the time**
  - Base 12 (for hours for instance)
  - Base 60 (for minutes for instance) 50 minutes and 33 seconds is a time counted in bases 60,
    It makes $50 \times 60 + 33 = 3033$ seconds
    btw: $60 = 5 \times 12$
- **In the 70's other bases were used to teach counting**
  - "Modern mathematics"

How do Martians count?[1]

- **We have seen numbers on the planet Mars**
  - $\equiv \int \cap \equiv \Theta$ or $\cap \cap \int \Theta$
- **We have found the way the Martians count:**

Θ	theta	0
$\int$	int	1
$\cap$	cap	2
$\equiv$	equiv	3
$\int \Theta$	int, theta	4
$\int \int$	int, int	5
$\int \cap$	int,cap	6
$\int \equiv$	int, equiv	7
$\cap\Theta$	cap, theta	8
$\cap \int$	cap, int	9
$\cap\cap$	cap, cap	10
$\cap \equiv$	cap, equiv	11
$\equiv \Theta$	equiv, theta	12
$\equiv \int$	equiv,int	13
$\equiv \cap$	equiv,cap	14
$\equiv\equiv$	equiv, equiv	15
$\int \Theta\Theta$	int, theta,theta	16

[1] source: Jeff Duntemann's book

Counting in Martian

- **Each symbol has a value**
  - Θ theta, 0
    $\int$, int, 1
    $\cap$, cap, 2
    $\equiv$, equiv, 3
  - Symbol *int* (int) is the unit
  - Symbol Θ (theta) is just a place holder
- **The value of a number depends on where a symbol is placed**
  - First column on the right: value of the symbol
  - Second column on the right: value of the symbol $\times \int \Theta$ (i.e. 4)
  - Third column : value of the symbol $\times \int \Theta\Theta$ (i.e. $16 = 4^2$)

Use Martian base

- **Meaning of $\cap \cap \int \Theta$**
  - Θ (theta) in the first column $= 0$
  - $\int$ (int) in the second column $= \int \times \int \Theta = 1 \times 4 = 4$
  - $\cap$ (cap) in the third column $= \cap \times \int \Theta\Theta = 2 \times 4^2 = 32$
  - $\cap$ (cap) in the fourth column $= \cap \times \int \Theta\Theta\Theta = 2 \times 4^3 = 2 \times 64 = 128$
  - $\cap \cap \int \Theta = \Theta + \int \times \int \Theta + \cap \times \int \Theta\Theta + \cap \times \int \Theta\Theta\Theta = 0 + 4 + 32 + 128 = 164$

Use Martian base

- **Meaning of $\equiv \int \equiv \cap \Theta$**
  - Θ (theta) in the first column $= 0$
  - $\cap$ (cap) in the second column $= \cap \times \int \Theta = 2 \times 4 = 8$
  - $\equiv$ (equiv) in the third column $= \equiv \times \int \Theta\Theta = 3 \times 4^2 = 3 \times 16 = 48$
  - *int* (int) in the fourth column $= \int \times \int \Theta\Theta\Theta = 1 \times 4^3 = 1 \times 64 = 64$
  - *equiv* (equiv) in the fifth column $= \equiv \times \int \Theta\Theta\Theta\Theta = 3 \times 4^4 = 3 \times 256 = 768$
  - $\cap\cap\int\Theta = \Theta + \int \times \int \Theta + \cap \times \int \Theta\Theta + \cap \times \int \Theta\Theta\Theta = 0 + 8 + 48 + 64 + 768 = 888$

Essence of a number base

- **Romans used a system where letters represented values**
  - MMXIV means 2014
  - MCMXC means 1990
  - The positions of the letters are not bounded to a column, but rather to their neighbors (for adding or substracting)
- **We use only columnar systems: the position of a number means the value**
  - in all bases 10 (or $\int \Theta$ for martians) represents the base
  - Number in column number 0 is multiplied by $base^0 = 1$
  - Number in column number 1 is multiplied by $base^1 = 10_{base}$
  - Number in column number 2 is multiplied by $base^2 = 100_{base}$
  - Number in column number 3 is multiplied by $base^3 = 1000_{base}$

Octal

Octal

- **Counting in octal**
  - 0, 1, 2 ,3 ,4 ,5, 6 , 7, 10
  - We do not use 8 and 9 anymore
  - 10 means 8
  - 11 means 9
  - 12 means 10
- **Octal uses base 8**
  - So the 8 does not exist!
  - 27 octal means $7 + 2 \times 8$

Octal table

0	zero	0
1	one	1
2	two	2
3	three	3
4	four	4
5	five	5
6	six	6
7	seven	7
10	ten octal	8
11	eleven octal	9
12	twelve octal	10
13	thirteen octal	11
14	fourteen oct.	12
15	fifteen oct.	13
16	sixteen oct.	14
17	seventeen oct.	15
20	twenty oct.	16

The octal numbers

- **Value of a number depends on its column**
  - A number in the unit column (column number 0) is just its value
  - 7 octal means 7
- **Column number one is multiplied by 8**
  - 10 octal means 8
  - 20 octal means 16
  - 70 octal means $7 \times 8 = 56$
- **Column number two is multiplied by 64**
  - 100 octal means 64

The powers of 8

- $1 octal = 8^0 = 1$
- $10_8 = 8^1 = 8$
- $100_8 = 8^2 = 64$
- $1\ 000_8 = 8^3 = 512$
- $1\ 0000_8 = 8^4 = 4096$
- $100\ 000_8 = 8^5 = 32\ 768$

Converting from octal into decimal

- **Suppose we have the number** 76225_8
  - $76225_8 = 70000_8 + 6000_8 + 200_8 + 20_8 + 5_8$
  - $5_8 = 5 \times 1 = 5$
  - $20_8 = 2 \times 10_8 = 2 \times 8^1 = 16$
  - $200_8 = 2 \times 100_8 = 2 \times 8^2 = 128$
  - $6000_8 = 6 \times 1000_8 = 6 \times 8^3 = 3072$
  - $70000_8 = 7 \times 10000_8 = 7 \times 8^4 = 28672$

Hexadecimal

Hexadecimal

- **Hexadecimal = base 16**
  - Is the real base for programmers
- **Digits**
  - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ...
  - A (10), B (11), C (12), D (13), E (14), F (15)
  - 16 is the base and therefore written 10 in Hexadecimal
- **Counting**
  - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 1F, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 2C, 2D, 2E, 2F,30
- **Notation**
  - In the remainder of this course, we will denote Hexadecimal numbers with a finishing H.
  - So 1 becomes 1H or 2F becomes 2FH

Hexadecimal table I

$0H$	zero	0
$1H$	one	1
$2H$	two	2
$3H$	three	3
$4H$	four	4
$5H$	five	5
$6H$	six	6
$7H$	seven	7
$8H$	eight	8
$9H$	nine	9
AH	A	10
BH	B	11
CH	C	12
DH	D	13
EH	E	14
FH	F	15
$10H$	One -oh hexadecimal	16
$11H$	One -one hexadecimal	17
$12H$	One -two hex.	18
$13H$	One -three hex.	19
$14H$	One -four hex.	20

Hexadecimal table II

$15H$	One -five hex.	21
$16H$	One -six hex.	22
$17H$	One -seven hex.	23
$18H$	One -eight hex.	24
$19H$	One -nine hex.	25
$1AH$	One -A hex.	26
$1BH$	One -B hex.	27
$1CH$	One -C hex.	28
$1DH$	One -D hex.	29
$1EH$	One -E hex.	30
$1FH$	One -F hex.	31
$20H$	Two -oh hex.	32

Table of powers of 16

- **Hexadecimal uses powers of 16**

$1H$	16^0	1
$10H$	16^1	16
$100H$	16^2	256
$1000H$	16^3	4096
$10000H$	16^4	65536
$100000H$	16^5	1048576
$1000000H$	16^6	16777216

Anatomy of a number

- **Let us evaluate the number** $3C0A9H$

$$
\begin{array}{rrrrrr}
 & & & & & 9H \\
 & & & & A & 0H \\
 & & & 0 & 0 & 0H \\
 & & C & 0 & 0 & 0H \\
+ & 3 & 0 & 0 & 0 & 0H \\
\hline
 & 3 & C & 0 & A & 9H
\end{array}
$$

- $3 \times 65536 + 12 \times 4096 + 0 \times 256 + 10 \times 16 + 9 \times 1$
- $196608 + 49152 + 0 + 160 + 9 = 245929$

Conversions

From Hex to Decimal

- **Method**
  - Compute the value of each column and add the results
- **Decimal value of** $7A2H$
  - add 2 (for the 2 in column 0)
  - add 10×16 (for the A in column 1)
  - add 7×256 (for the 7 in column 2)

Other example

- **Value of** $C6F0DBH$
  - $B \times 1 = 11$
  - $D \times 16 = 13 \times 16 = 208$
  - $0 \times 256 = 0$
  - $F \times 4096 = 15 \times 4096 = 61\ 440$
  - $6 \times 65\ 536 = 393\ 216$
  - $C \times 1\ 048\ 576 = 12 \times 1\ 048\ 576 = 12\ 582\ 912$
- **Total = 13 037 787**

From Decimal to Hex

- **We want to write 449 in hex.**
  - Find the largest hex column value that is contained at least once in 449
    4096 is to large, and 256 is perfect.
  - find which number of 256 goes into 449 (remember 5rd Grade division)
  - $449/256 = 1.7539$ so 1 is the lefmost hex digit
  - Let us substract 1×256 from 449, we obtain 193
  - The next power of 16 is 16 itself, how many times 16 goes into 193
  - $193/16 = 12.0625$, so C is the next hex digit
  - $193 - 12 * 16 = 1$ the remainder is one and the next value of 16 (i.e. 16^0) is 1, so the next hex digit is 1
- **The hex value of 449 is** $1C1H$

Another example

- **What is the hex value of** 988 664
  - The largest power of 16 contained in the number is 65 536
  - 65 536 goes 15 times in 988 664, so the left most hex digit is **F**.
  - The remainder is $988\,664 - 65\,536 \times 15 = 5624$
  - The next power of 16 is 4096, which goes only once in 5624. So the next hex digit is **1**.
  - The remainder is $5624 - 4096 = 1528$
  - The next power of 16 is 256, 256 goes 5 times into 1528
  - The next hex digit is **5**
  - The remainder is $1528 - 5 * 256 = 248$
  - The next of 16 is 16 itself, 16 goes 15 times into 148
  - The next hex digit is **F**
  - The remainder is $148 - 16 \times 15 = 8$
  - The last hex digit is **8**
- **The hex value is** $F15F8H$

Arithmetic in hexadecimal

Arithmetic in Hex

- **You need to do arithmetic directly in Hex**
  - Conversion to decimal and back from it will be impossible
  - for instance to add CH and FH,
  - CH is 12, FH is 15, $CH + FH$ is 27
  - then we convert 27 back into hex: $1BH$
- **Need to learn additions by heart**
  - Use flash cards for instance

Additions in Hex

$$\begin{array}{r} 9 \\ +1 \\ \hline 0AH \end{array} \quad \begin{array}{r} 8 \\ +2 \\ \hline 0AH \end{array} \quad \begin{array}{r} 7 \\ +3 \\ \hline 0AH \end{array} \quad \begin{array}{r} 6 \\ +4 \\ \hline 0AH \end{array} \quad \begin{array}{r} 5 \\ +5 \\ \hline 0AH \end{array}$$

$$\begin{array}{r} A \\ +1 \\ \hline 0BH \end{array} \quad \begin{array}{r} 9 \\ +2 \\ \hline 0BH \end{array} \quad \begin{array}{r} 8 \\ +3 \\ \hline 0BH \end{array} \quad \begin{array}{r} 7 \\ +4 \\ \hline 0BH \end{array} \quad \begin{array}{r} 6 \\ +5 \\ \hline 0BH \end{array}$$

$$\begin{array}{r} B \\ +1 \\ \hline 0CH \end{array} \quad \begin{array}{r} A \\ +2 \\ \hline 0CH \end{array} \quad \begin{array}{r} 9 \\ +3 \\ \hline 0CH \end{array} \quad \begin{array}{r} 8 \\ +4 \\ \hline 0CH \end{array} \quad \begin{array}{r} 7 \\ +5 \\ \hline 0CH \end{array} \quad \begin{array}{r} 6 \\ +6 \\ \hline 0CH \end{array}$$

$$\begin{array}{r} C \\ +1 \\ \hline 0DH \end{array} \quad \begin{array}{r} B \\ +2 \\ \hline 0DH \end{array} \quad \begin{array}{r} A \\ +3 \\ \hline 0DH \end{array} \quad \begin{array}{r} 9 \\ +4 \\ \hline 0DH \end{array} \quad \begin{array}{r} 8 \\ +5 \\ \hline 0DH \end{array} \quad \begin{array}{r} 7 \\ +6 \\ \hline 0DH \end{array}$$

Additions in Hex II

$$\begin{array}{rrrrrrr} D & C & B & A & 9 & 8 & 7 \\ +1 & +2 & +3 & +4 & +5 & +6 & +7 \\ \hline 0EH & 0EH & 0EH & 0EH & 0EH & 0EH & 0EH \end{array}$$

$$\begin{array}{rrrrrrr} E & D & C & B & A & 9 & 8 \\ +1 & +2 & +3 & +4 & +5 & +6 & +7 \\ \hline 0FH & 0FH & 0FH & 0FH & 0FH & 0FH & 0FH \end{array}$$

$$\begin{array}{rrrrrrrr} F & E & D & C & B & A & 9 & 8 \\ +1 & +2 & +3 & +4 & +5 & +6 & +7 & +8 \\ \hline 10H & 10H & 10H & 10H & 10H & 10H & 10H & 10H \end{array}$$

$$\begin{array}{rrrrrrr} F & E & D & C & B & A & 9 \\ +2 & +3 & +4 & +5 & +6 & +7 & +8 \\ \hline 11H & 11H & 11H & 11H & 11H & 11H & 11H \end{array}$$

Additions in Hex III

$$\begin{array}{r} F \\ +3 \\ \hline 12H \end{array} \quad \begin{array}{r} E \\ +4 \\ \hline 12H \end{array} \quad \begin{array}{r} D \\ +5 \\ \hline 12H \end{array} \quad \begin{array}{r} C \\ +6 \\ \hline 12H \end{array} \quad \begin{array}{r} B \\ +7 \\ \hline 12H \end{array} \quad \begin{array}{r} A \\ +8 \\ \hline 12H \end{array} \quad \begin{array}{r} 9 \\ +9 \\ \hline 12H \end{array}$$

$$\begin{array}{r} F \\ +4 \\ \hline 13H \end{array} \quad \begin{array}{r} E \\ +5 \\ \hline 13H \end{array} \quad \begin{array}{r} D \\ +6 \\ \hline 13H \end{array} \quad \begin{array}{r} C \\ +7 \\ \hline 13H \end{array} \quad \begin{array}{r} B \\ +8 \\ \hline 13H \end{array} \quad \begin{array}{r} A \\ +9 \\ \hline 13H \end{array}$$

$$\begin{array}{r} F \\ +5 \\ \hline 14H \end{array} \quad \begin{array}{r} E \\ +6 \\ \hline 14H \end{array} \quad \begin{array}{r} D \\ +7 \\ \hline 14H \end{array} \quad \begin{array}{r} C \\ +8 \\ \hline 14H \end{array} \quad \begin{array}{r} B \\ +9 \\ \hline 14H \end{array} \quad \begin{array}{r} A \\ +A \\ \hline 14H \end{array}$$

$$\begin{array}{r} F \\ +6 \\ \hline 15H \end{array} \quad \begin{array}{r} E \\ +7 \\ \hline 15H \end{array} \quad \begin{array}{r} D \\ +8 \\ \hline 15H \end{array} \quad \begin{array}{r} C \\ +9 \\ \hline 15H \end{array} \quad \begin{array}{r} B \\ +A \\ \hline 15H \end{array}$$

Additions in Hex IV

$$\begin{array}{r} F \\ +7 \\ \hline 16H \end{array} \quad \begin{array}{r} E \\ +8 \\ \hline 16H \end{array} \quad \begin{array}{r} D \\ +9 \\ \hline 16H \end{array} \quad \begin{array}{r} C \\ +A \\ \hline 16H \end{array} \quad \begin{array}{r} B \\ +B \\ \hline 16H \end{array}$$

$$\begin{array}{r} F \\ +8 \\ \hline 17H \end{array} \quad \begin{array}{r} E \\ +9 \\ \hline 17H \end{array} \quad \begin{array}{r} D \\ +A \\ \hline 17H \end{array} \quad \begin{array}{r} C \\ +B \\ \hline 17H \end{array}$$

$$\begin{array}{r} F \\ +9 \\ \hline 18H \end{array} \quad \begin{array}{r} E \\ +A \\ \hline 18H \end{array} \quad \begin{array}{r} D \\ +B \\ \hline 18H \end{array} \quad \begin{array}{r} C \\ +C \\ \hline 18H \end{array}$$

$$\begin{array}{r} F \\ +A \\ \hline 19H \end{array} \quad \begin{array}{r} E \\ +B \\ \hline 19H \end{array} \quad \begin{array}{r} D \\ +C \\ \hline 19H \end{array}$$

Additions in Hex V

$$\begin{array}{r} F \\ +B \\ \hline 1AH \end{array} \quad \begin{array}{r} E \\ +C \\ \hline 1AH \end{array} \quad \begin{array}{r} D \\ +D \\ \hline 1AH \end{array}$$

$$\begin{array}{r} F \\ +C \\ \hline 1BH \end{array} \quad \begin{array}{r} E \\ +D \\ \hline 1BH \end{array}$$

$$\begin{array}{r} F \\ +D \\ \hline 1CH \end{array} \quad \begin{array}{r} E \\ +E \\ \hline 1CH \end{array}$$

$$\begin{array}{r} F \\ +E \\ \hline 1DH \end{array} \qquad \begin{array}{r} F \\ +F \\ \hline 1EH \end{array}$$

Columns and carries

- **Method for adding hex numbers**
- **Add each column starting from the right and carry into the next column anytime the result exceeds** $0FH$

$$
\begin{array}{ccccccc}
 & 1 & & & & 1 & \\
 & 2 & F & 3 & 1 & A & DH \\
+ & 9 & 6 & B & A & 0 & 7H \\
\hline
 & C & 5 & E & B & B & 4H
\end{array}
$$

- The most you can carry is 1

Substraction and borrows

- **We have to mentally reverse**
  - if $E + 6 = 14H$ then $14H - 6 = E$
- **We have to substract column by column**
  - Start from right

$$
\begin{array}{rcccc}
 & F & 7 & 6 & C & H \\
- & A & 0 & 5 & B & H \\
\hline
 & 5 & 7 & 1 & 1 & H
\end{array}
$$

Borrows

- **Need for borrows if a value to substract is larger than the one we substract from.**
  - $9 - A = ???$

$$\begin{array}{rccc} & 9 & 2 & H \\ - & 4 & F & H \\ \hline & ? & ? & \end{array}$$

  - We need to add 10H (i.e. 16_{10} to the number for the substraction to be possible.

$$\begin{array}{rccc} & 9 & 2 & H \\ - & 4_1 & F & H \\ \hline & 4 & 3 & H \end{array}$$

Borrows accross Multiple Columns

- **We may have to transfer the borrow accross more than one column**
- $$\begin{array}{rccccc} & F & 0 & 0 & 0 & H \\ - & 3 & B & 6 & C & H \\ \hline & ? & ? & ? & ? & H \end{array}$$
- $$\begin{array}{rccccc} & F & 0 & 0 & 0 & H \\ - & 3 & B & 6_1 & C & H \\ \hline & ? & ? & ? & 4 & H \end{array}$$
- $$\begin{array}{rccccc} & F & 0 & 0 & 0 & H \\ - & 3 & B_1 & 6_1 & C & H \\ \hline & ? & ? & 9 & 4 & H \end{array}$$

Borrows accross Multiple Columns

- $$\begin{array}{rccccc} & F & 0 & 0 & 0 & H \\ - & 3_1 & B_1 & 6_1 & C & H \\ \hline & ? & 4 & 9 & 4 & H \end{array}$$
- $$\begin{array}{rccccc} & F & 0 & 0 & 0 & H \\ - & 3_1 & B_1 & 6_1 & C & H \\ \hline & B & 4 & 9 & 4 & H \end{array}$$

Binary

Binary

- **There are only two digits (0 and 1) in the base**
- **Each column has a value two times the column to its right**
- **Counting**

  0
  1
  10
  11
  100
  101
  110
  111
  1000
  1001
  1010

Powers of 2

Binary	Power of 2	decimal
1	2^0	1
10	2^1	2
100	2^2	4
1000	2^3	8
10000	2^4	16
100000	2^5	32
1000000	2^6	64
10000000	2^7	128
100000000	2^8	256
1000000000	2^9	512
10000000000	2^{10}	1024
100000000000	2^{11}	2048
1000000000000	2^{12}	4096
10000000000000	2^{13}	8192
100000000000000	2^{14}	16384

Notation

- **Values in binary should be noted with a B**
  $110B$ means 6
  whereas
  $110H$ means 272
  and 110 means 110
- **Notations in scientific books use subscript**
  110_2 means 6_{10}
  whereas
  110_{16} means 272_{10}
  and 110_{10} means 110_{10}
- **But it is not usable inside source files or simple text editors.**

Why are computer binary?

- **Other machines have been tested with base 3**
  - 1840 Thomas Fowler built a ternary calculating machine from wood
  - 1958 Nikolay Brusentsov (USSR) built the *Setun* computer
  - in 1973 he built an enhanced version called *Setun-70*
  - In the USA, a computer was built in 1973 *Ternac*
- **Because lights are either on or off**
  - In an electrical device : voltage is present or not
  - It means 1 or 0

Hex as shorthand for binary

Hex as shorthand for binary

- 218 **is expressed in binary:** $11011010B$
- **expressed in hex:** DAH
  - AH (or $0AH$ in assembler) represents the number 10
  - Conversion in Binary: $1010B$
  - They are the last four digits of 218 in binary
  - DH is also $1101B$

218		decimal
1101	1010	binary
D	A	hex

Hex as shorthand for binary

- **If we have a 32 binary number**
  $11110000000000001111101001101110B$
- We can split it into group of 4
  $11110000000000001111101001101110$
- Each group of 4 is represented by one Hex value

1111	0000	0000	0000	1111	1010	0110	1110
F	0	0	0	F	A	6	E

- **The hex equivalent is** $F000FA6EH$

Conclusion

- **Computers work only in binary**
  - Notations in Binary are too long
  - We use hex to represent binary values
- **You should be familiar with hex notation**
  - It is the center of assembler
  - One solution: do the exercises!

Source

- **Book: Assembly Language Step by Step (3rd edition)**
  by Jeff Duntemann